>>> Automatic complexity of monotone Boolean functions

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Date: May 24, 2018

>>> Recognize this?

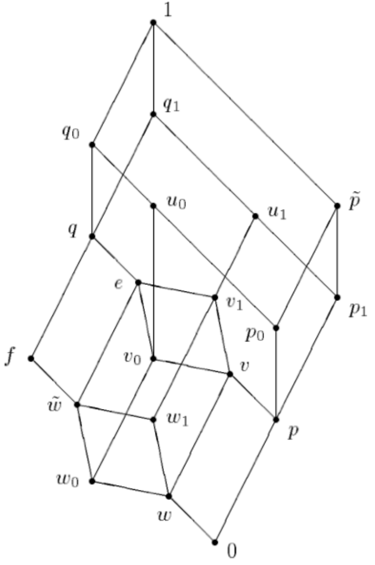
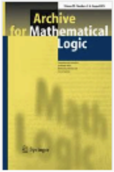


Figure 1: Lattice L_{20}

[~]$ _

>>> Lattice theory and computability theory



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The strength of the Grätzer-Schmidt theorem

Authors Authors and affiliations

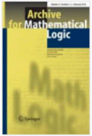
Katie Brodhead, Mushfeq Khan, Bjørn Kjos-Hanssen, William A. Lampe, Paul Kim Long V. Nguyen, Richard A. S

Article
First Online: 05 May 2016

Abstract

The Grätzer-Schmidt theorem of lattice theory states that each algebraic lattice is isom
the congruence lattice of an algebra. We study the reverse mathematics of this theorem
also show that

1. the set of indices of computable lattices that are complete is Π^1_1-complete;



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February 2014, Volume 53, Issue 1–2, pp 1–10 | Cite as

Polynomial clone reducibility

Authors   Authors and affiliations

Quinn Culver

Article
First Online: 11 September 2013

Abstract

Polynomial clone reducibilities are generalizations of the truth-table reducibilities. A polynomial clone is a set of functions over a finite set X that is closed under composition and contains all the constant and projection functions. For a fixed polynomial clone $\mathscr{C}$, a sequence $B \in X^\omega$ is $\mathscr{C}$ -reducible to $A \in X^\omega$ if there is an algorithm that computes B from A using only effectively selected functions from $\mathscr{C}$. We show that if A is Kurtz random and $\mathscr{C}_1 \nsubseteq \mathscr{C}_2$ are polynomial clones, then there is a B that is $\mathscr{C}_1$ -reducible to A but not $\mathscr{C}_2$ -reducible to A. This implies a generalization of a result first proved by Lachlan (Z Math Logik Grundlagen Math 11:17–44, 1965) for the case $|X| = 2$. We also show that the same result holds if Kurtz random is replaced by Kolmogorov–Loveland stochastic.

>>> Lattice theory and computability theory

Complexity of index sets of computable lattices

Nguyen, Paul Kim Long Vu. University of Hawai'i at Manoa, ProQuest Dissertations Publishing, 2014. 3648578.

Full text - PDF | Preview - PDF | Abstract/Details | References 12

3648578.pdf 6 / 44

ABSTRACT

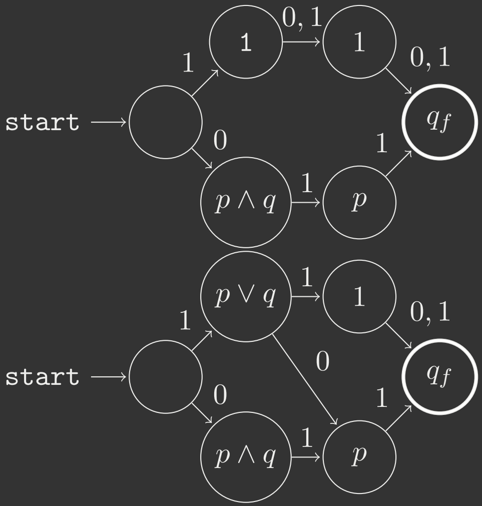
We analyze computable algebras in the sense of universal algebra and the index set complexity of properties of such algebras. We look at the difficulty of determining properties of **Con** (**A**), the congruence lattice of an algebra **A**. In particular, we introduce the notion of a class of algebras *witnessing* the complexity of a property of algebras and show that computable lattices witness the Π^0_2-completeness of being simple, as well as witnessing the Σ^0_3-completeness of having finitely many congruences. Finally, in our main result, we show that the property "to be subdirectly irreducible" is Σ^0_3-complete as well, and in the process show that computable lattices witness this.

[~]$ _

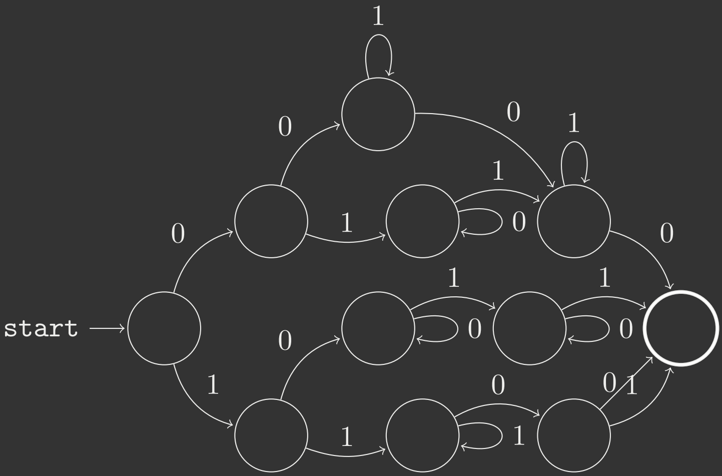
* Order/lattice theory and automata theory
* Automata theory and finance

Exercise 1.8 (Asian option). Consider the three-period model of Example 1.2.1, with $S_0 = 4$, $u = 2$, $d = \frac{1}{2}$, and take the interest rate $r = \frac{1}{4}$, so that $\tilde{p} = \tilde{q} = \frac{1}{2}$. For $n = 0, 1, 2, 3$, define $Y_n = \sum_{k=0}^{n} S_k$ to be the sum of the stock prices between times zero and n. Consider an *Asian call option* that expires at time threc and has strike $K = 4$ (i.e., whose payoff at time three is $\left(\frac{1}{4}Y_3 - 4\right)^+$). This is like a European call, except the payoff of the option is based on the average stock price rather than the final stock price. Let $v_n(s, y)$

>>> Monotone examples

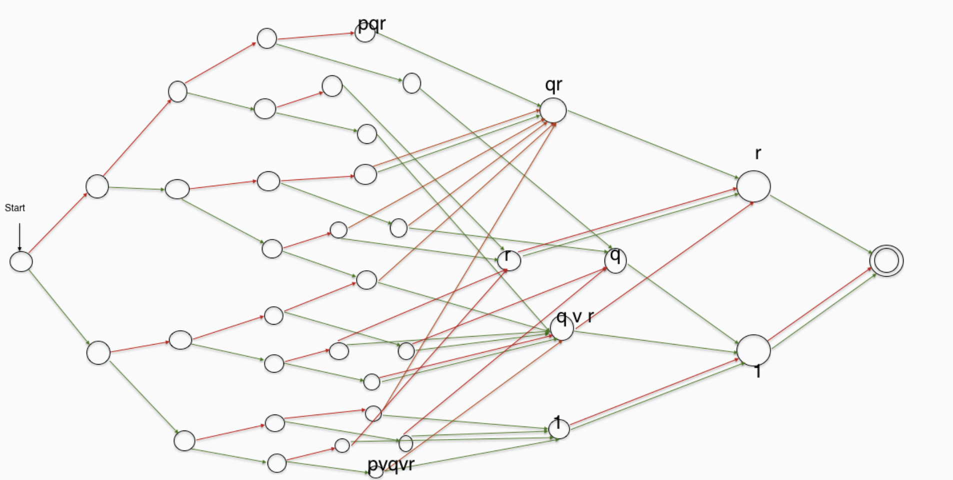


Asian option, and European call option.

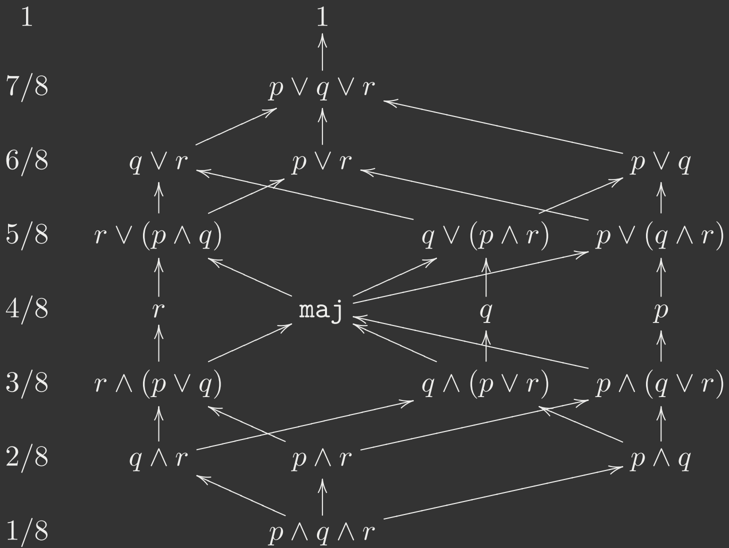


A minimal automaton for $\{0000, 0100, 1011, 1100, 1101\}$.

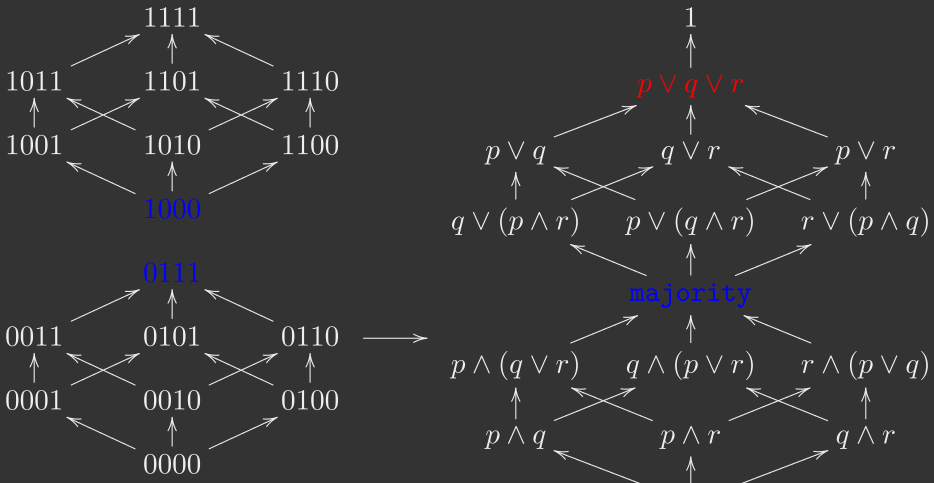
>>> Complexity 38 set for $n = 7$ (39 is max for monotone sets)



>>> F_3^-, nonzero monotone Boolean functions of 3 variables



>>> Isotone map from $\mathbf{2}^4$ to F_3^-, almost 1:1



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>>> Surprise?
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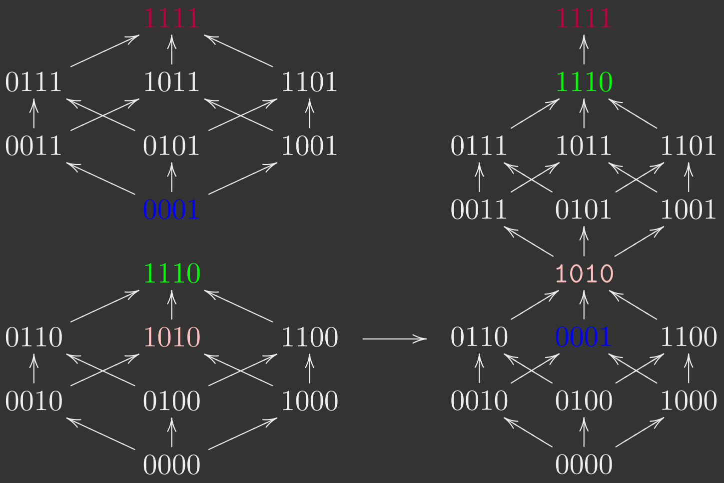
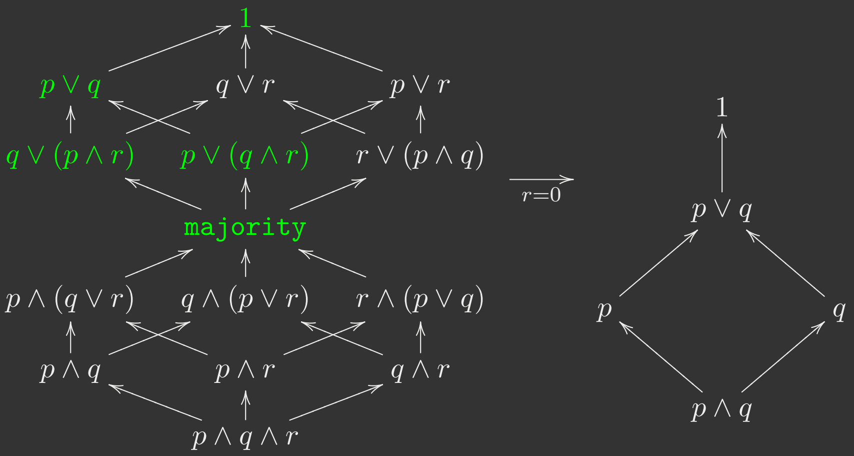
Isotone 1:1 map from $\mathbf{2}^4$ to F_3^-



>>> Adequacy



1
p ∨ q
q ∨ r
p ∨ r
q ∨ (p ∧ r)
p ∨ (q ∧ r)
r ∨ (p ∧ q)
majority
p ∧ (q ∨ r)
q ∧ (p ∨ r)
r ∧ (p ∨ q)
p ∧ q
p ∧ r
q ∧ r
p ∧ q ∧ r
r=0
1
p ∨ q
p
q
p ∧ q

>>> Question

Is there an isotone 1:1 map from $\mathbf{2}^7$ to F_4^-?
(Yes, for $\mathbf{2}^6$.)
Is there an adequate one?