Hawaiian Excursions into Equational Logic

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Algebras and Lattices in Hawai'i
Honoring Ralph Freese, Bill Lampe, and JB Nation
Manoa, HI
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Bosses

Bosses

Party

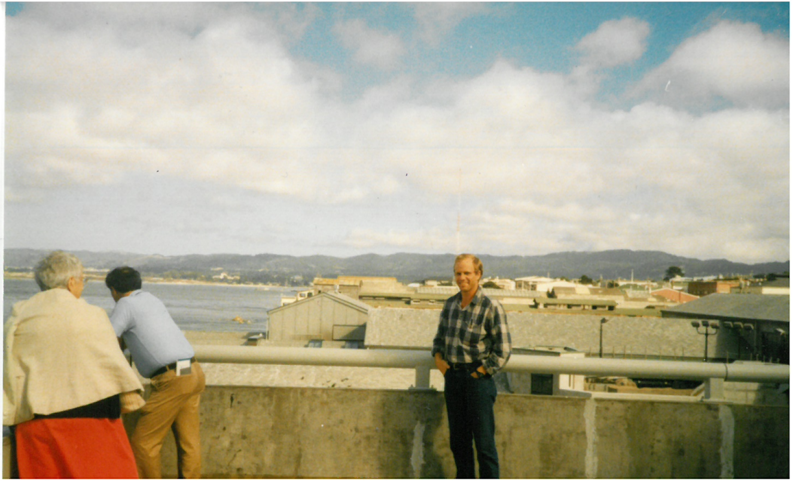
Party!

A Fantastic Theorem

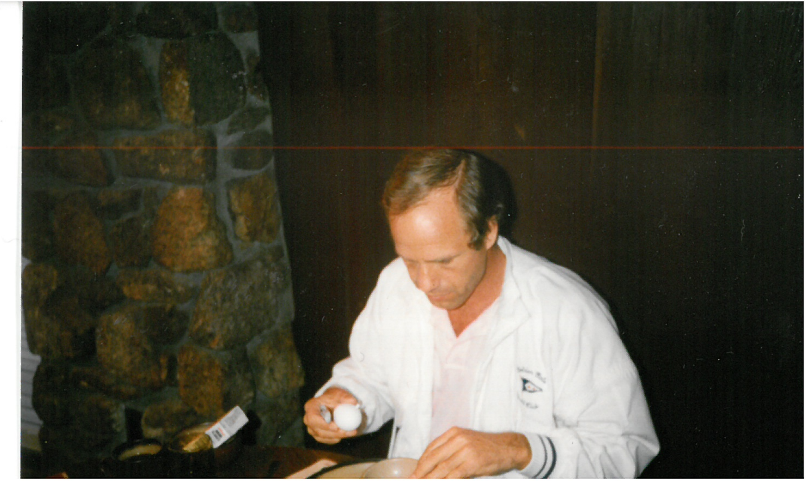
Ralph Proved This

The equational theory of modular lattices is undecidable.

Working is Hard



Sustenance Makes Theorems!



Lattices? Ralph Knows Some Other Stuff

Some Other Island with Volcanoes.

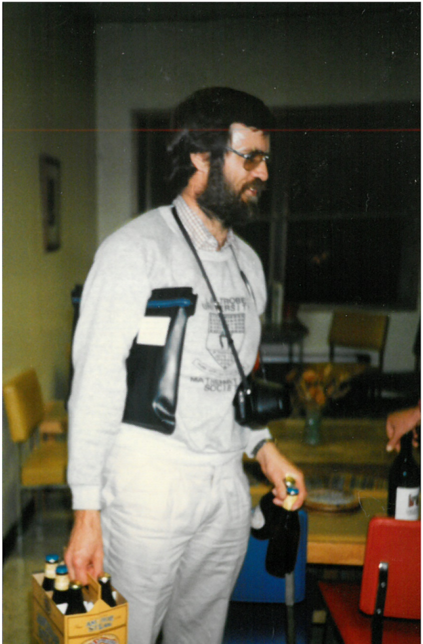
Something Fishy

What a Deal!

The Evidence

Interlude: What is That Guy Eating?

Interlude: My Lattice Got Tight!



Interlude: Our Friends

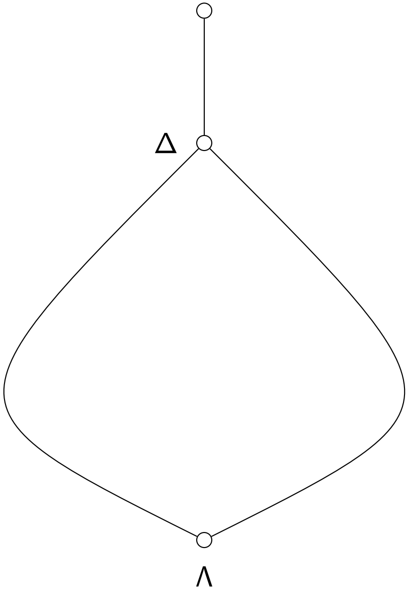
JB and Max on That Other Island

A Break for Some Mathematics

Equational theories (in the signature of lattices) form a lattice. Those that lie above the equational theory Λ of all lattices form another lattice. This lattice is algebraic and the finitely based equational theories of lattices are the compact elements.

What Does This Lattice Look Like??

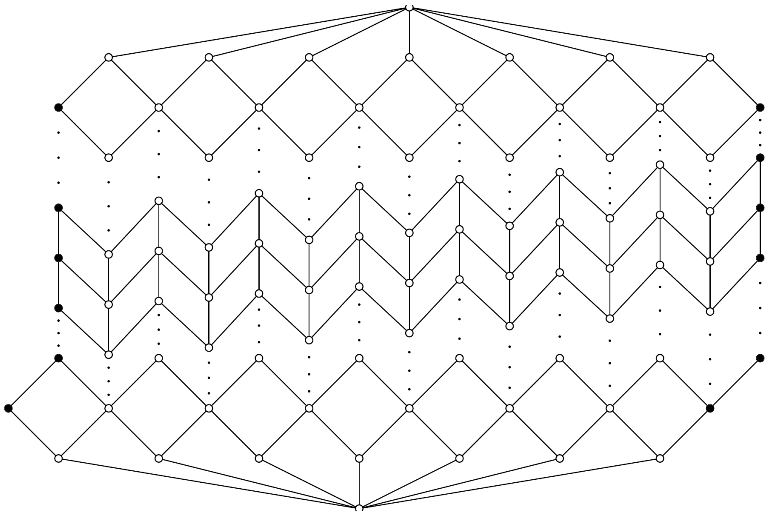
The Lattice of Equational Theories of Lattices



The Finite Depth Conjecture

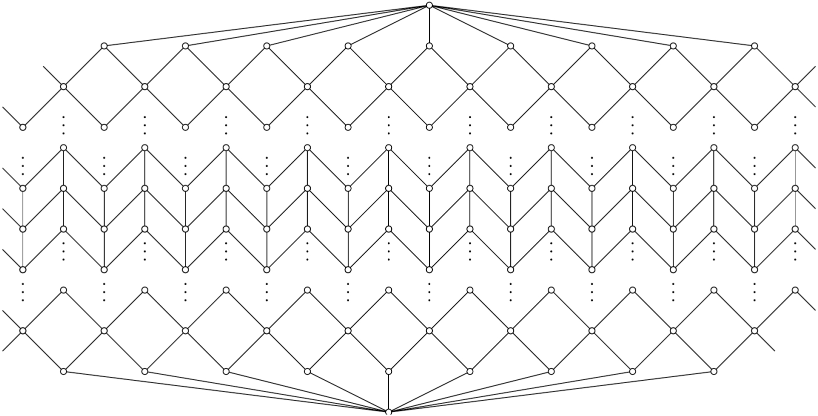
Every equational theory of lattices that is of finite depth is the equational theory of a finite lattice.

JB Says No!



The Lattice $(\mathbf{J} \star \mathbf{B})_2$

This Lattice is Inherently Nonfinitely Based



The Lattice $\mathbf{J} \star \mathbf{B}$

Ralph, JB, and George say so.

Here is a Problem or Two

A variety fails to be locally finite *in the finite sense* provided there is a natural number p so that the variety contains arbitrarily large finite p-generated algebras.

A locally finite variety $\mathcal{V}$ of finite signature is inherently nonfinitely based *in the finite sense* if and only if $\mathcal{V}^{(n)}$ fails to be locally finite in the finite sense for every natural number n.

Is $\mathbf{J} \star \mathbf{B}$ inherently nonfinitely based in the finite sense?

Is there as inherently nonfinitely based group?

Where is that professional photographer?

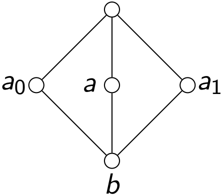
More Hard Work

The Riemann Hypothesis doesn't have a chance

Lampe's Zipper

Bill Lampe's Zipper Condition

Let $\mathbf{L}$ be an algebraic lattice. We say that $\mathbf{L}$ satisfies the **zipper condition** provided whenever I is a nonempty set, $a, b \in L$ and $a_i \in L$ for all $i \in I$ such that $\bigvee_{i \in I} a_i = 1$ and $a \wedge a_i = b$ for all $i \in I$, then $a = b$.

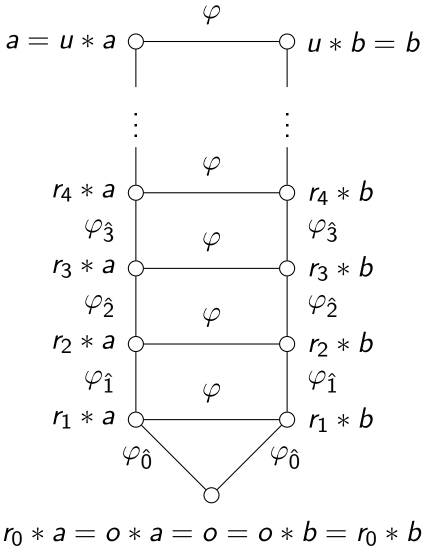


The Lattice $\mathbf{M}_3$ Fails the Zipper Condition

Bill Lampe's Zipper Theorem

Any principal filter in the lattice of all equational theories of some signature satisfies the zipper condition.

A Hint to the Proof



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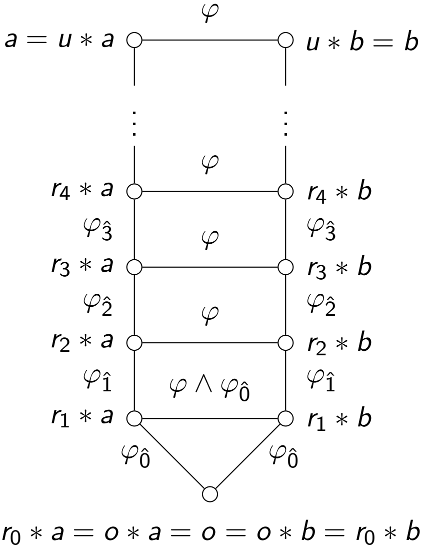

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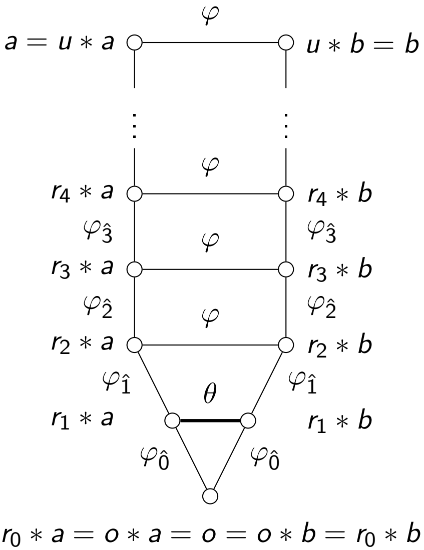

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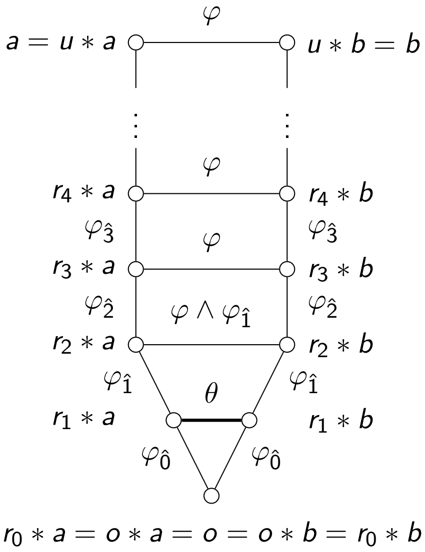

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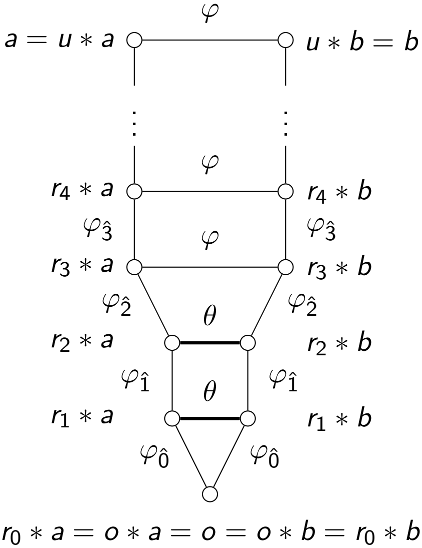

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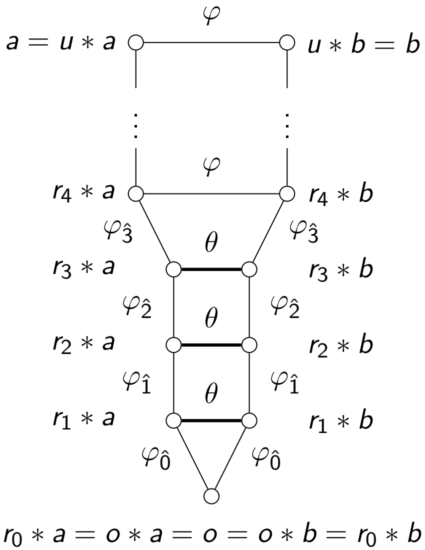

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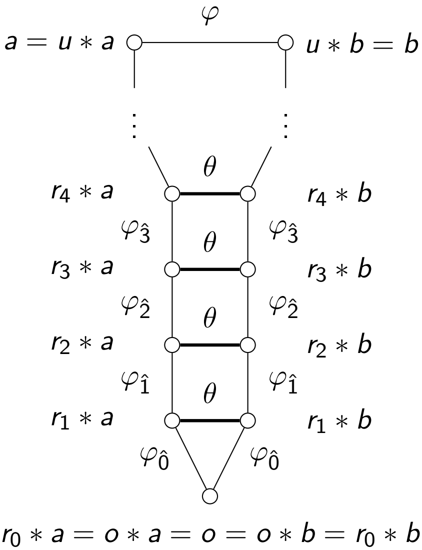

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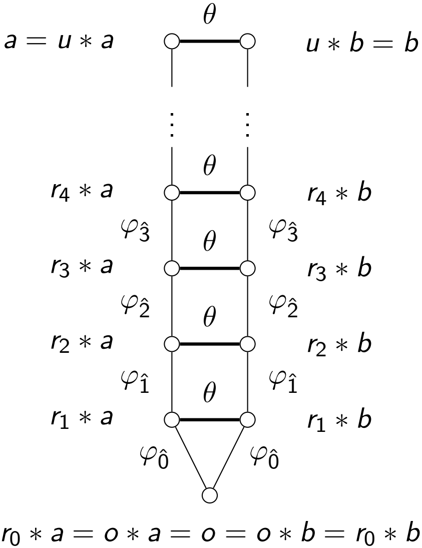
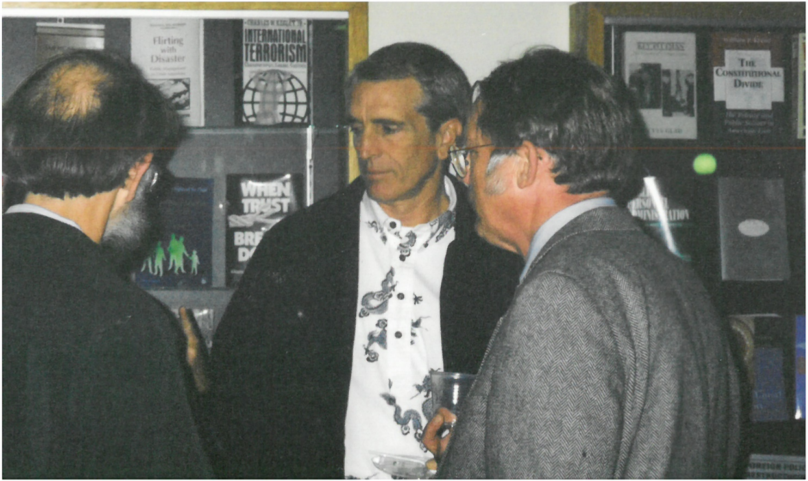


ISN'T THAT A ZIPPY PROOF?!

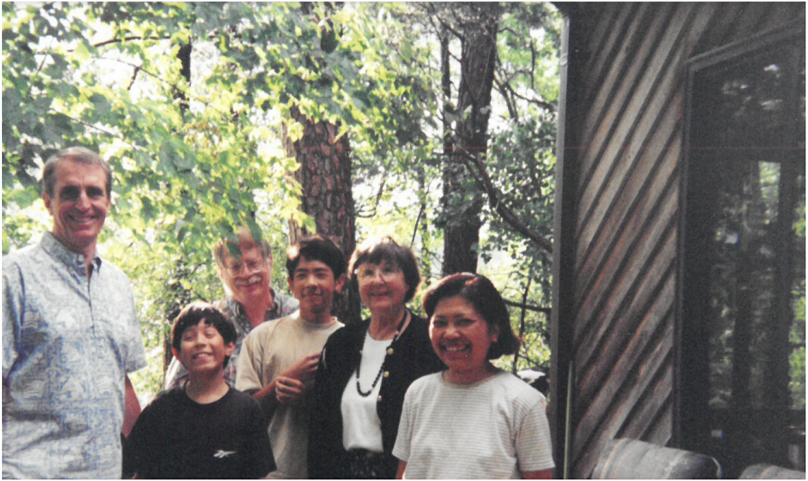
Who knows where this is?

What's so surprising?



What are those guys discussing?

Bill promised to tell me how to grow tall

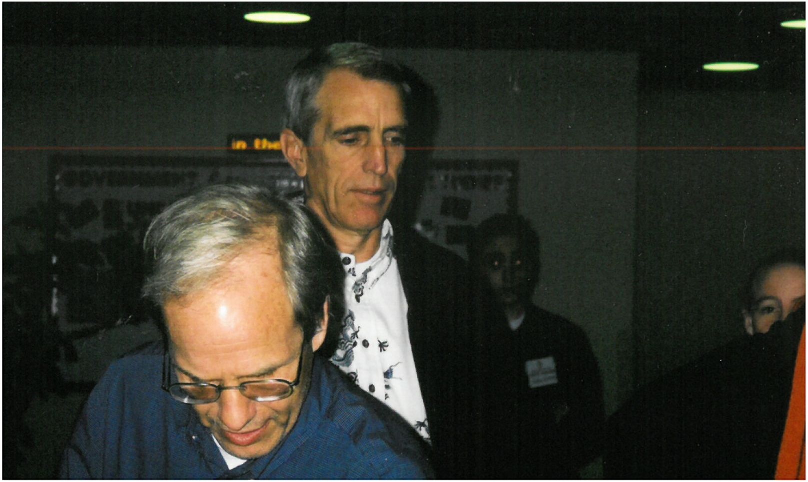


That little guy is 32 now!

Alden is just shy

The usual suspects

The unusual suspects



I’ll help in case you get into trouble with the cork.

Show me those koalas!

NOT a classroom!

A Tajik Tea House—is the really tea, Keith?

All that jazz!

All we ever do

Another Problem

Let Δ be a finite signature with at least two unary operation symbols or at least one operation symbol of rank at least 2. Let $\mathcal{L}_\Delta$ be the lattice of all equational theories of the signature Δ

Hilbert's Tenth Problem for $\mathcal{L}_\Delta$

Is there an algorithm that, upon input of a finite set of equations in the language of lattice theory, will determine whether the set of equations has a solution in $\mathcal{L}_\Delta$?

Of course, Hilbert did not pose this problem. Rather he posed the problem in which the ring $\langle \mathbb{Z}, +, \cdot, -, 0, 1 \rangle$ of integers replaces the lattice $\mathcal{L}_\Delta$.

Do you believe in Volume II?